Calc BC Series Gold Solutions

① (E)

② $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$

$= \left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + \left(\frac{1}{4}-\frac{1}{5}\right) + \ldots$

$= 1$ (B) (This is called a Telescopic Series)

③ $r = -\frac{1}{2}$

$S_n = \frac{a}{1-r}$

$= \frac{2}{1-\left(-\frac{1}{2}\right)}$

$= \frac{2}{\frac{3}{2}}$

$= \frac{4}{3}$ (A)

④ (B)

⑤ (A)

⑥ (A) through (D) are P-Series which diverge, (E)

⑦ (C)

(8) $\lim_{n\to\infty} \frac{n}{\sqrt{4n^2-1}}$

$= \lim_{n\to\infty} \frac{n}{\sqrt{(2n-1)^2}} = \frac{1}{2} \neq 0$

diverges by n^{th} term test (D)

(9) (C)

(10) $\lim_{n\to\infty} \frac{n-1}{n+1} = 1$

Diverges (B)

(11) (B)

(12) $S = \frac{\frac{2}{3}}{1-\frac{2}{3}}$

$= 2$ (D)

(13) (E)

(14) (A)

(15) $\sum_{n=1}^{\infty} \frac{x^n}{n}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{|x|^{n+1}}{n+1} \cdot \frac{n}{|x|^n}$

$= |x|$

$|x| < 1 \implies -1 < x < 1$

Endpoints

$x=1$: $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

$x=-1$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges A.S.T.

Ioc: $-1 \leq x < 1$ (C)

(16) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+1)^n}{n!}$

$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{|x+1|^{n+1}}{(n+1)!} \cdot \frac{n!}{|x+1|^n}$

$= \lim_{n\to\infty} \frac{|x+1|}{n+1}$

$= 0$, converges $\forall x$ (A)

(17) $\lim_{n\to\infty} \frac{(n+1)! \, |x-3|^{n+1}}{n! \cdot |x-3|^n}$

$= \lim_{n\to\infty} (n+1)|x-3|$

$= \begin{cases} 0, & x = 3 \quad \text{converges} \\ \infty, & x \neq 3 \quad \text{diverges} \end{cases}$ (C)

(18) $(x-2) + \frac{(x-2)^2}{4} + \frac{(x-2)^3}{9} + \dots$

$$1 + \frac{x-2}{2} + \frac{(x-2)^2}{3} + \frac{(x-2)^3}{4} + \dots \frac{(x-2)^n}{n+1} + \dots = \sum_{n=1}^{\infty}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{|x-2|^{n+1}}{n+2} \cdot \frac{n+1}{|x-2|^n}$$

$$= |x-2|$$

$$|x-2| < 1 \implies -1 < x-2 < 1$$

$$1 < x < 3$$

Endpoints

$x=1$: $\sum \frac{(-1)^n}{n+1}$ converges by AST

$x=3$ $\sum \frac{(1)^n}{n+1}$ diverges by LCT w/ $\sum \frac{1}{n}$

$1 \le x < 3$ (B)

(19) $\int_0^x f(t)\,dt = \int_0^x t^n\,dt$ $\qquad f(x) = \sum_{n=0}^{\infty} x^n$

$$= \frac{t^{n+1}}{n+1}\Big|_0^x = \frac{x^{n+1}}{n+1}$$ $\qquad$ IOC : $x \in (-1,1)$

endpoints $x=-1$: converges by AST

$x=1$: Diverges

IOC $\implies -1 \le x < 1$ (D)

(20) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$

$\frac{\left(-\frac{1}{2}x\right)^4}{4!} = \frac{1}{16 \cdot 4!} = \frac{1}{16 \cdot 24} = \frac{1}{384}$ E

(21) $f(x) = (1+x)^{1/2}$ $\quad f(0) = 1$

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$ $\quad f'(0) = \frac{1}{2}$

$f''(x) = -\frac{1}{4}(1+x)^{-3/2}$ $\quad f''(0) = -\frac{1}{4}$

$f'''(x) = \frac{3}{8}(1+x)^{-5/2}$ $\quad f'''(0) = \frac{3}{8}$

$f(x) \approx 1 + \frac{1}{2}x - \frac{1}{4} \cdot \frac{x^2}{2!} + \frac{3}{8} \cdot \frac{x^3}{3!}$

$= 1 + \frac{1}{2}x - \frac{x^2}{8} + \frac{1}{16}x^3$ B

(22) $e^x = e + e(x-1) + \frac{e(x-1)^2}{2!} + \frac{e(x-1)^3}{3!} + \ldots$

$= e\left(1 + (x-1) + \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3!}\right.$

$= e \sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}$ D

(23) $\frac{f'''(a)}{3!}(x-a)^3$

$f(x) = \cos x$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$f'''(x) = \sin x$

$f'''\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

Coeff: $\frac{\frac{\sqrt{2}}{2}}{3!} = \frac{\sqrt{2}}{2} \cdot \frac{1}{6} = \frac{\sqrt{2}}{12} = \frac{1}{6\sqrt{2}}$ D

(25) $f(x) = e^{\sin x}$

$f'(x) = \cos x \cdot e^{\sin x}$

$f''(x) = \cos^2 x \cdot e^{\sin x} - \sin x \, e^{\sin x}$

Coeff. $\frac{f''(a)}{2!} = \frac{1}{2!}$ C

(26) $f(x) = x \cdot \ln x$

$f'(x) = x \cdot \frac{1}{x} + \ln x = 1 + \ln x$

$f''(x) = 1 + \frac{1}{x}$

$f'''(x) = -\frac{1}{x^2}$

$f^{(4)}(x) = 2x^{-3}$

$f^{(5)}(x) = -6x^{-4}$

Coeff $= \frac{f^{(5)}(1)}{5!}$

$= \frac{-6}{5!}$

$= \frac{-6}{120}$

$= -\frac{1}{20}$ A

(27) less than next term

$\leq \frac{|x|^5}{5!}$

$\leq \frac{1}{5!}$

$\leq .0083$ (E)

(28) $f(x) = e^{-x}$

$f(.2) = e^{-.2}$

$= .8187$ (C)

(29) geometric w/

$r = \frac{\pi^3}{3^\pi}$

$$S_n = \frac{\frac{\pi^3}{3^\pi}}{1 - \frac{\pi^3}{3^\pi}}$$

$$= \frac{\pi^3}{\cancel{3^\pi}} \cdot \frac{\cancel{3^\pi}}{3^\pi - \pi^3}$$

(D)

(30) error $< u_{301}$

$< \frac{1}{902}$

$< .0011$ (A)

(31) $\tan^{-1} 1 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{(-1)^n}{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

$\frac{1}{2n+1} < \frac{1}{1000}$

$2n+1 > 1000$

$2n > 999$

$n > 499.5 \Rightarrow 500$ (E)

(32) $f(x) = (1+x)^p$

$f(0) = 1$

$f'(x) = p(1+x)^{p-1}$; $f'(0) = p$

$f''(x) = p(p-1)(1+x)^{p-2}$; $f''(0) = p(p-1)$

$f'''(x) = p(p-1)(p-2)(1+x)^{p-3}$; $f'''(0) = p(p-1)(p-2)$

$$(1+x)^p \approx P_3(x) = 1 + px + \frac{p(p-1)x^2}{2!} + \frac{p(p-1)(p-2)}{3!}x^3$$

(C)

(33) $\frac{1}{1+x} = \frac{1}{1-(-x)}$

$= 1 - x + x^2 - x^3 + x^4 - \dots$

$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots$

$\ln(1+2x) = 2x - \frac{1}{2}(2x)^2 + \frac{1}{3}(2x)^3 - \dots$ (A)

(34) $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{(n+1)\, 2^{n+1}}{|x|^{n+1}} \cdot \frac{|x|^n}{n \cdot 2^n}$

$= \lim_{n\to\infty} \frac{2}{|x|}$

$= \frac{2}{|x|}$

$\frac{2}{|x|} < 1$

$\frac{|x|}{2} > 1$

$|x| > 2$

$x = -2$: Diverges

$x = 2$: Diverges

$|x| > 2$

D